division of Tortoises is confined to Aldabra and never extended to the Mascarenes proper, and that the Tortoises from the latter islands can be externally, though not osteologically, distinguished *as a whole* from the Galapagos Tortoises, as will be seen from the following synopsis:—

I. Nuchal plate absent. Frontal portion of the skull flat. Fourth cervical vertebra biconvex. Pelvis with broad symphysial bridge.

A. Gular plate double; sternum of moderate extent ..... GALAPAGOS TORTOISES.

B. Gular plate single; sternum short ..... MASCARENE TORTOISES.

a. Carapace thin, thickened towards the margins; centre of the last vertebral plate raised into a hump, which is separated from the penultimate vertebral by a transverse depression: *Tortoises of Mauritius* (*T. triserrata*, *T. inepta*, *T. indica*, *T. leptocnemis*).

b. The entire carapace extremely thin and fragile, all the bones very slender: *Tortoise of Rodriguez* (*T. Vosmœri*).

II. Nuchal plate present. Frontal portion of the skull convex. Third cervical vertebra biconvex. Pelvis with narrow symphysial bridge. Gular plate double. Carapace thick. ALDABRA TORTOISES (*T. elephantina*, *T. Daudinii*, *T. ponderosa*, *T. hololissa*).

II. "On certain Definite Integrals." By W. H. L. RUSSELL, F.R.S. Received December 5, 1876.

The following paper is a continuation of one recently inserted in the 'Proceedings of the Royal Society.'

$$(13.)\quad \int_0^{\frac{\pi}{2}} d\theta \log_\epsilon (1+2\alpha\cos^2\theta\cos 2\theta+\alpha^2\cos^4\theta)=\pi\log_\epsilon\frac{\alpha+4}{4}.$$

$$(14.)\quad \int_0^{\infty} \frac{d\theta\sqrt{1+\alpha\cos\theta+\sqrt{1+2\alpha\cos\theta+\alpha^2}}}{1+\theta^2} = \frac{\pi}{\sqrt{2}}\cdot\frac{\sqrt{\epsilon+\alpha}}{\sqrt{\epsilon}}.$$

Similarly we may find

$$(15.)\quad \int_0^{\infty} d\theta\sqrt{\cos 2\theta+\mu\cos 3\theta+\sqrt{1+2\mu\cos\theta+\mu}}\log_\epsilon\frac{\theta^2+\epsilon^2}{\theta^2+\alpha^2}.$$

$$(16.)\quad \int_0^{\frac{\pi}{2}} \frac{\log_\epsilon\cos\theta\cos 2\theta}{1-2x^2\cos 4\theta+x^4} = \frac{\pi}{8x(1-x^2)}\log_\epsilon\frac{1+x}{1-x}.$$

$$(17.)\quad \int_0^{\pi} \theta d\theta\,.\,\frac{\sin\theta+x^2\sin 3\theta}{1-2x^2\cos 4\theta+x^4} = \frac{\pi}{\sqrt{x}}\left\{\log_\epsilon\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)^{\frac{1}{4}}+\frac{1}{2}\tan^{-1}\sqrt{x}\right\}.$$

$$(18.)\quad \int_0^{\frac{\pi}{2}} \theta d\theta \frac{\sin\theta\cos\theta}{1+(2a-a^2)\cos^2\theta+a^2(a^2+2a+4)\cos^4\theta} = \frac{\pi}{(a^2+2a)\sqrt{3}} \tan^{-1}\frac{a\sqrt{3}}{a+4}.$$

$$(19.)\quad \int_0^{\pi} \frac{\epsilon^{x\cos\theta}\left(\epsilon^{x\cos\theta}\sin\theta+\sin(\theta+x\sin\theta)\right)\theta d\theta}{\epsilon^{2x\cos\theta}+2\epsilon^{x\cos\theta}\cos(x\sin\theta)+1} = \frac{\pi}{x}\log_\epsilon\frac{2\epsilon^x}{\epsilon^x+1}.$$

$$(20.)\quad \int_0^{\pi} \frac{\epsilon^{x\cos\theta}\left(\epsilon^{x\cos\theta}\sin\theta+\sin(\theta+x\sin\theta)\right)d\theta}{\epsilon^{2x\cos\theta}+2\epsilon^{x\cos\theta}\cos(x\sin\theta)+1} = 1.$$

$$(21.)\quad \int_0^{\frac{\pi}{2}} \cos\theta d\theta \frac{\cos(\theta+\tan\theta)+a\cos\theta}{1+2a\cos\theta+a^2} = \frac{\pi}{4(\epsilon+a)}.$$

$$(22.)\quad \int_0^{\frac{\pi}{2}} \cos\theta d\theta \frac{\cos(\theta+c\tan\theta)+a\cos\theta\cos c\tan\theta}{\sin^2\theta+(1+2a+a^2)\cos^2\theta} = \frac{\pi}{2(a+2)\epsilon^c}.$$

$$(23.)\quad \int_0^{\frac{\pi}{2}} \frac{d\theta}{\cos\theta}\cdot\frac{\cos(\theta-\tan\theta)+a\cos\theta\cos\tan\theta}{\sin^2\theta+(1+2a+a^2)\cos^2\theta} = \frac{\pi}{\epsilon^{a+1}}.$$

$$(24.)\quad \int_0^{\infty} \frac{dx(\epsilon^{x\cos\alpha}\cos\alpha\sin\alpha+\mu)}{\epsilon^{2x\cos\alpha}+2\mu\epsilon^{x\cos\alpha}\cos\alpha\sin\alpha+\mu^2} = \frac{\cos\alpha}{\mu}\log_\epsilon(1+\mu).$$

$$(25.)\quad \int_0^{\infty} \frac{dx\epsilon^{x\cos\alpha}\sin(x\sin\alpha)}{\epsilon^{2x\cos\alpha}+2\mu\epsilon^{x\cos\alpha}\cos\alpha\sin\alpha+\mu^2} = \frac{\sin\alpha}{\mu}\log_\epsilon(1+\mu).$$

$$(26.)\quad \int_0^{\infty} \frac{(\epsilon^{2x\cos\alpha}+\epsilon^{x\cos\alpha})\cos(x\sin\alpha)dx}{\epsilon^{4x\cos\alpha}+2\epsilon^{2x\cos\alpha}\cos(2x\sin\alpha)+1} = \frac{\pi}{4}\cos\alpha.$$

$$(27.)\quad \int_0^{\infty} \frac{(\epsilon^{3x\cos\alpha}+\epsilon^{x\cos\alpha})\sin(x\sin\alpha)dx}{\epsilon^{4x\cos\alpha}+2\epsilon^{2x\cos\alpha}\cos(2x\sin\alpha)+1} = \frac{\pi}{4}\sin\alpha.$$

$$(28.)\quad \int_0^{\pi} \theta d\theta\sin\theta\cos^{n-m-1}\theta\frac{1-\cos^{2n}\theta}{1-\cos^{2n}\theta} = \frac{\pi^2}{2n}\tan\frac{m\pi}{2n}.$$

$$(29.)\quad \int_0^{\pi} \theta d\theta\sin\theta\cos^{n-m-1}\theta\frac{1-\cos^{2m}\theta}{1-\cos^{n}\theta} = \frac{\pi}{m}-\frac{\pi^2}{n}\operatorname{cotan}\frac{m\pi}{n}.$$

$$(30.)\quad \int_0^{\frac{\pi}{2}} \frac{d\theta\cos^n\theta(\cos n\theta+\mu\cos^n\theta)}{(1+2\mu\cos^n\theta\cos n\theta+\mu^2\cos^{2n}\theta)(a^2\sin^2\theta+b^2\cos^2\theta)} = \frac{\pi}{2ab}\cdot\frac{a^n}{(a+b)^n+\mu a^n}.$$

$$(31.)\quad \int_0^{\pi} d\theta \epsilon^{\sin 2\theta \cos 2\theta} \cos(\sin^2\theta \sin 2\theta) = \frac{\pi}{\sqrt[4]{\epsilon}}.$$

$$(32.)\quad \int_0^{\pi} d\theta \sin\theta \epsilon^{\sin 2\theta \cos 2\theta} \sin(\sin^2\theta \sin 2\theta + \theta) = \frac{\pi}{2\sqrt[4]{\epsilon}}.$$

$$(33.)\quad \int_0^{\frac{\pi}{2}} d\theta\theta \{\epsilon^{\alpha \sin 2\theta} \sin 2(\theta - \alpha\cos^2\theta) + \epsilon^{-\alpha \sin 2\theta} \sin 2(\theta + \alpha\cos^2\theta)\} = \frac{\pi \sin\alpha}{2\alpha}.$$

$$(34.)\quad \int_0^{\omega} \frac{(x - a^2x)dx}{\left((1+a+a^2)\sin ax - a\sin 3ax\right)(1+x^2)} = \pi \frac{\epsilon^{2a}+a}{\epsilon^{2a}-a} \cdot \frac{\epsilon^a}{\epsilon^{2a}-1}.$$

$$(35.)\quad \int_0^{\omega} \frac{(x - a^2x)dx}{\left((1+a+a^2)\cos ax + a\cos 3ax\right)(1+w^2)} = \pi \frac{\epsilon^{2a}-a}{\epsilon^{2a}+a} \cdot \frac{\epsilon^a}{\epsilon^{2a}+1}.$$

$$(36.)\quad \int_0^{\pi} d\theta \epsilon^{x\cos\theta} \cos(x\sin\theta + \theta) \log_\epsilon(1+\cos\theta) = \frac{\pi(\epsilon^x - 1)}{\epsilon^x x}.$$

$$(37.)\quad \int_0^{\frac{\pi}{2}} \frac{d\theta \log\ (1 + 2a\cos^n\theta \cos n\theta + a^2\cos^{2n}\theta)}{a^2\sin^2\theta + b^2\cos^2\theta} = \frac{\pi}{ab} \log_\epsilon \frac{(a+b)^n + aa^n}{(a+b)^n}.$$

I observe that in (28) n and m must be odd and even or *vice versâ*, and that in (29) n must be even and m odd.

The integrals here are not solitary examples: each of them may be considered as the representative of a very numerous class. I have selected as examples those which seemed most suitable to be brought before the Royal Society.

P.S.—The following were added 13th February, 1877 :—

$$(38.)\quad \int_0^{\pi} \theta d\theta\, \epsilon^{x\cos\theta} \sin\left(x\sin\theta + \frac{3\theta}{2}\right) \cos\frac{\theta}{2} = \frac{\pi}{2}\left(\frac{x-1}{x^2} + \frac{1}{\epsilon^x x^2}\right).$$

$$(39.)\quad \int_0^{\pi} \theta d\theta \frac{\sin 2\theta + (1-x)\sin\theta}{1 - 2x\cos\theta + x^2} = \pi\left\{\frac{(x+1)\log_\epsilon(1+x) - x}{x^2}\right\}.$$